Determination of 2D-tectonic deformations using affine transformation

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Summary
Presented in this paper is the determination of 2D-tectonic movements of parts of the Earth's surface using affine transformation. The proposed method is tested on a geodetic network which covers a hazardous region »Mirovo« which lies north east of Bulgaria.

Zusammenfassung
In der vorliegenden Arbeit wird die Bestimmung von zweidimensionalen tektonischen Bewegungen von Teilen der Erdoberfläche mit Hilfe von affinen Transformationen vorgestellt. Die vorgeschlagene Methode wird anhand eines geodätischen Netzes getestet, das ein erdbebengefährdetes Gebiet im Nordosten Bulgariens überdeckt.

1 Introduction

Some solid bodies, including blocks of rocks of the Earth's surface, behave as elastic bodies. According to Hook's law, the ideally elastic body is subjected to uniform deformations, which are infinitesimal and recoverable. The uniform deformation is one and the same in all points of the body, regardless of its magnitude.

The concept in the basis of analysis of the Earth's surface tectonic deformations has an entirely geometric character. This concept is developed in the theory of elasticity in the continuous media mechanics and in geodesy – in the theory of affine transformation.

In the deformation analysis of geodetic measurements, the investigated object is most often idealized by a certain number of points, connected between them in a geodetic network. The geodetic network is observed in the epoch t^0 (zero measurement) and in the epochs t^i (repeated measurements, $i = 1, 2, \dots n$), and it is adjusted within the framework of free network. The results from the single free network adjustment should not be compared together because the movements of the observed points can cause displacement, rotation and scale difference in the network. This will lead to incorrect interpretation of the coordinate differences.

It is appropriate to adjust the measurements from the *i*-th repeated observation to the network of the zero measurement using the methods of the geodetic transformations in order to make a correct analysis of the point movements in the network.

The theoretical basis of all investigations in the present work is the assumption that a linear dependence exists between the position of the points in the network in the initial epoch and in the phase of already occurred deformations.

2 Homogeneous kinematic model (Affine transformation)

It is known that the objective evaluation of the geo-kinematic parameters should be invariant with respect to the change of the reference system. The assessment of the *Fixed point method* from the viewpoint of *Datum problem* (Design 0 range) is already presented by Grafarend et al. (1979) using the theory of Gauss-Markov-Model ($GMM = \{\boldsymbol{I}, \boldsymbol{A}, \boldsymbol{x}, \boldsymbol{Q}\}$).

It has been proven that there is no formal difference between the determinations of the unknown vector $\boldsymbol{x}(\partial x, \partial y)$ and the differential vector $\boldsymbol{d}(u, v)$ in one *GMM*

$$\begin{aligned} dx(x, y) &= u(x, y) \\ dy(x, y) &= v(x, y) \end{aligned} \tag{1}$$

for free network adjustment of the geodetic network.

In this way the solutions of the *Fixed point method* can be interpreted as the solutions of a *GMM*. These solutions will be shown in a model of homogeneous kinematics (Affine transformation).

If the *deformation vector* of the points during the strained state of the body is designated by x', and the radius vector of the same points in the initial (non-strained) state is designated by x, then a linear relationship of the following type exists between x' and x:

$$\mathrm{x'} = \mathrm{Fx} + \mathrm{t} \tag{2}$$

where F is the gradient of deformation and t is translation vector. The gradient of deformation F is obtained through

$$\boldsymbol{F} = \boldsymbol{RV} \tag{3}$$

respectively $\mathrm{F} = d\mathrm{R} + \mathrm{V}$ (if we apply infinitely small deformation) from an antisymetric rotation tensor $\boldsymbol{R}$ given by

$$d\mathrm{R} = \begin{Vmatrix} 0 & dw \\ -dw & 0 \end{Vmatrix}, \tag{4}$$

where dw is a rotation angle and V a symmetric stretch tensor

$$\boldsymbol{V} = \boldsymbol{I} + \boldsymbol{E}, \tag{5}$$

with a unit matrix I and the strain tensor E given by

$$\boldsymbol{E} = \begin{Vmatrix} e_{xx} & e_{xy} \\ e_{yx} & e_{yy} \end{Vmatrix} = \boldsymbol{R}_{\Theta}\, \boldsymbol{E}_{\lambda}\, \boldsymbol{R}_{\Theta}^{T}, \tag{6}$$

where Q is orientation of the main axis and

$$\boldsymbol{E}_{\lambda} = \begin{Vmatrix} e_1 & 0 \\ 0 & e_2 \end{Vmatrix}. \tag{7}$$

The solution above can be interpreted as a kinematic as well as a geometrical one, in the sense of the least square solution.

3 The calculation of pure deformation (strain) by affine transformation

The case is considered when 2D-deformations of a network of simplex-triangles are determined. Then, using simplex-models in a system of orthogonal Cartesian coordinates, it is assumed that each element is subjected to uniform deformation. It is also supposed that both the components of the strain tensor and the gradients of deformation are constant within the boundaries of the element.

If the projections along the X and Y axes of the displacement vector for the triangle vertices i, j, k are designated respectively as U_i, V_i, U_j, V_j, U_k, V_k, then the dependencies will be valid for the simplex model

$$\begin{aligned} U_i &= X_i^{'} - X_i = \alpha + e_{xx} X_i + e_{xy} Y_i \\ V_i &= Y_i^{'} - Y_i = \beta + e_{yx} X_i + e_{yy} Y_i \end{aligned} \tag{8}$$

where e_{11}, e_{12}, e_{21}, e_{22}, α, β are coefficients that are constant within the boundaries of each triangular element.

From (8) one obtains

$$\begin{aligned} X_i^{'} &= \alpha + (e_{xx} + 1) X_i + e_{xy} Y_i \\ Y_i^{'} &= \beta + e_{yx} X_i + (e_{yy} + 1) Y_i \end{aligned} \tag{9}$$

From the affine transformation

$$\begin{aligned} X_i^{'} &= a_1 + a_2 X_i + a_3 Y_i \\ Y_i^{'} &= b_1 + b_2 X_i + b_3 Y_i \end{aligned} \tag{10}$$

and (9) it follows that $\alpha = a_1$; $\beta = b_1$

$$\begin{aligned} a_2 &= (e_{xx} + 1), \quad \text{or} \quad e_{xx} = (a_2 - 1) \\ a_3 &= e_{xy}, \quad \text{or} \quad e_{xy} = a_3 \\ b_2 &= e_{yx}, \quad \text{or} \quad e_{yx} = b_2 \\ b_3 &= (e_{yy} + 1), \quad \text{or} \quad e_{yy} = (b_3 - 1)\,. \end{aligned} \tag{11}$$

The dependence

$$\gamma_{xy} = e_{xy} + e_{yx} \tag{12}$$

leads to

$$\gamma_{xy} = b_2 + a_3\,. \tag{13}$$

We then obtain for the principle strain axes

$$E_{1,2} = \frac{1}{2}\left[(a_2 + b_3 - 2) \pm \sqrt{(a_2 - b_3)^2 - 2(b_2 + a_3)^2}\right] \tag{14}$$

where

$$\begin{aligned} E_1 &= E_{\max} \\ E_2 &= E_{\min} \end{aligned} \tag{15}$$

The true bearing φ_E of the major axis e_{xx} is obtained from

$$tg2\varphi_E = \frac{b_2 + a_3}{a_2 - b_3} \tag{16}$$

$$\varphi_E = \frac{1}{2} arctg \frac{b_2 + a_3}{a_2 - b_3}\,. \tag{17}$$

4 Numerical Example

A hazard region in Northeast Bulgaria monitored by the »Mirovo« geodetic network, has been chosen for testing the proposed method, the network being divided into triangles (Fig. 1).

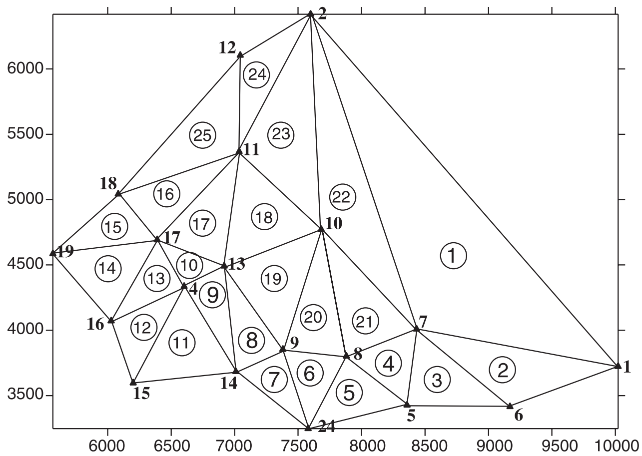

Fig. 1: Triangle Scheme of the »Mirovo« geodetic network

The measurements are made by »Geoprecise Engineering Ltd., Sofia, Bulgaria«. Using the developed software in the *Turbo Pascal* algorithmic language, the homogeneous tectonic deformations of the blocks of the Earth's surface have been determined. Eng. G. Iliev developed the program. The results are presented in Table 1 and in the corresponding graphic supplements (Fig. 2, Fig. 3 and Fig. 4).

No of Triangle	e_{xx}	e_{yy}	$e_{xx} - e_{yy}$
1	−0.0175	0.1109	−0.1284
2	−0.0181	−0.0549	0.0368
3	−0.0077	0.0720	−0.0797
4	−0.0287	0.0568	−0.0854
5	−0.0122	−0.0480	0.0384
6	−0.0274	−0.0341	0.0666
7	0.1752	0.0598	0.1154
8	−0.0403	0.3283	−0.3686
9	0.9838	0.3285	0.6554
10	0.3331	−0.0862	0.4192
11	0.2226	0.7638	−0.5411
12	−0.0686	0.3806	−0.4492
13	−0.1093	0.3299	−0.4393
14	0.1070	0.2691	−0.1622
15	0.7903	0.5748	0.2155
16	0.1734	0.5992	−0.3558
17	0.1245	0.2730	−0.1485
18	−0.0144	0.2907	−0.3051
19	−0.0803	0.2883	−0.3686
20	−0.3532	0.0711	−0.4243
21	0.1003	0.0588	0.0415
22	0.1429	−0.0915	0.2344
23	0.3377	−0.0947	0.4324
24	0.3567	1.1594	−0.8027
25	0.9467	1.0296	−0.0829

Table 1: Calculated 2D-tectonic deformations, »Mirovo« Geodetic Network

5 Concluding remarks

The following conclusions can be made on the basis of the investigation

- The proposed method can be used for determining 2D-tectonic deformations.
- The determination of the so-called *relative deformations* is correct only for linear changes of the displacements inside the accepted elementary figures, which are triangles in the case under consideration. The size of these triangles should be determined by the size of the expected deformations.
- The so-called *section line* is clearly seen in the graphic plots of the angular deformations, i. e. the sign is changed in the southern part of the site.
- The investigation of the displacements along the coordinate axes *dx* and *dy* shows that more intensive movements of the Earth's surface occur in the southern part of the site.

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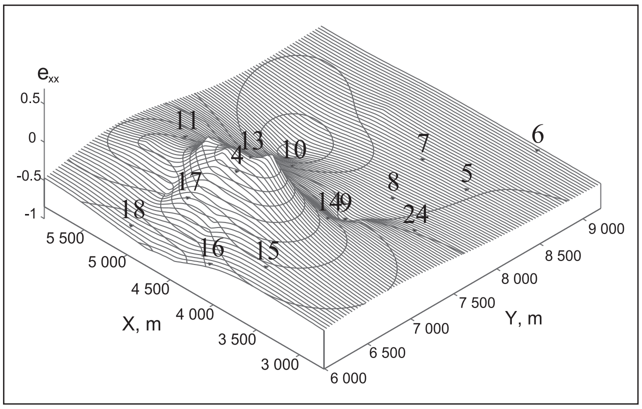

Fig. 2: Deformations along the X axis

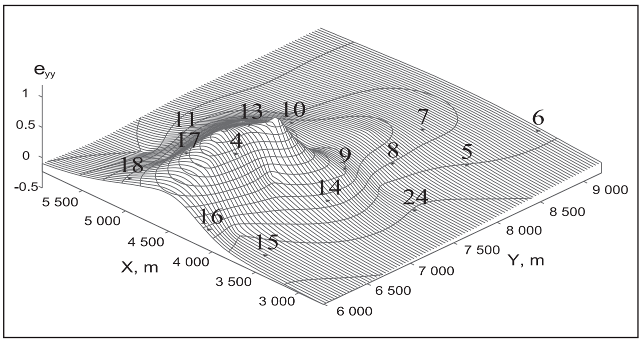

Fig. 3: Deformations along the Y axis

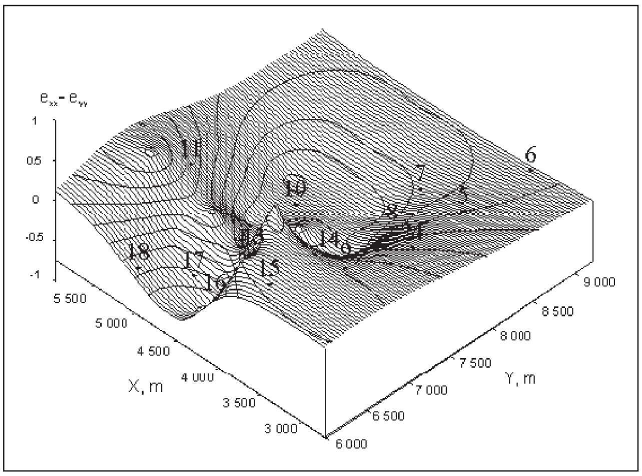

Fig. 4: Angular deformations $e_{xx} - e_{yy}$

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