# Improving the Performance of Alignment Processes of Inertial Measurement Units Utilizing Adaptive Pre-Filtering Methodology

Aboelmagd Noureldin, Eun-Hwan Shin and Naser El-Sheimy

#### Summary

The determination of the initial attitude angles of any inertial platform before starting its motion usually depends on Kalman filtering to provide fine alignment. The estimation accuracy of the attitude angles is limited by the relatively low signal to noise ratio (SNR) of the inertial sensors, especially for the gyroscopes. This article suggests the utilization of a finite impulse response (FIR) filter at the output of each inertial sensor to improve their SNR before using their measurement in estimating the initial tilt and heading angles. The filter parameters are adapted to their optimal values using the least mean square (LMS) criterion with known Earth rotation rate and Earth gravity components as the reference signals. The traditional coarse and fine alignment procedures are then employed for the computation of the three attitude angles. The filter design procedure requires absolutely no accelerations during the whole alignment. The suggested pre-filtering method has shown a significant impact on the accuracy of alignment procedures of inertial measurement units, which is very essential in providing precise inertial positioning for different navigation applications.

# Zusammenfassung

Die Bestimmung von anfänglichen Orientierungsparametern (Attitude Parameter) jeder inertialen Plattform vor dem Bewegungsstart hängt gewöhnlich vom Kalman-Filter ab, der das Feinalignment vornimmt. Die Schätzungsgenauigkeit der Orientierungsparameter ist dabei beschränkt durch das relativ niedrige Signal-Rausch-Verhältnis (SNR) der inertialen Sensoren, besonders aber der Kreisel. Dieser Beitrag schlägt die Anwendung von Filtern mit endlicher Impulsübertragung (FIR) für jeden der inertialen Sensoren vor, um das Signal-Rausch-Verhältnis zu verbessern, bevor die Messungen zur Schätzung der Orientierungparameter und des »Headings« verwendet werden. Die Filterparameter sind angepasst an ihre optimalen Werte mittels Kleinste-Quadrate-Ausgleichung und bekannter Erdrotationsund Erdschwerefeldkomponenten. Die traditionellen Grob- und Feinalignment-Methoden werden dann angewendet, um die drei Orientierungswinkel zu bestimmen. Es wird gezeigt, dass die vorgeschlagene Filtermethode einen großen Einfluss auf die Genauigkeit der Alignmentmethoden von inertialen Messsystemen (IMU) hat, was wesentlich für präzise inertiale Positionsbestimmung in den verschiedensten Navigationsanwendungen ist.

# 1 Introduction

The computational algorithm of a strapdown inertial navigation system (SINS) is based on a set of first order

differential equations for the position, the velocity and the attitude components of a moving platform (Titterton and Weston 1997). The successful utilization of the SINS algorithm requires accurate knowledge of the initial values of the above navigation parameters. The position initialization including the platform latitude, longitude and altitude is determined from external source (e. g. GPS). The tilt and heading initialization is known by alignment and is usually performed within the SINS using special computational operations on the inertial sensor measurements (Salychev 1998). This article focuses on modifying the alignment procedure with adaptive pre-filtering method to improve its performance.

## 1.1 Conventional Alignment Procedure

The purpose of the alignment procedure is to give initial estimate of the transformation matrix  $\mathbf{R}_{h}^{\ell}$  between the SINS body frame (b-frame) and the local-level frame (\ell-frame) (Schwarz and Wei 1990). The b-frame is defined by three mutually orthogonal axes along the transverse (x-axis), the forward (y-axis) and the vertical (z-axis) directions. On the other hand, the  $\ell$ -frame is defined by the east (x-axis), the north (y-axis) and upward (z-axis) directions. The  $\mathbf{R}_{b}^{\ell}$  matrix is then used to transform the inertial sensor measurements (specific force and angular velocity measurements) from the b-frame to the ℓ-frame. The alignment procedure is, generally, performed while the system is stationary and it consists of two stages (Salychev 1998). The first stage is the coarse alignment and the second stage is the fine alignment. During the coarse alignment, the accelerometers and the gyroscopes monitor the components of the Earth's gravity field and the Earth's rotation, respectively. These measurements are averaged over an interval of few minutes to provide a preliminary estimate of the three attitude angles (pitch, roll and azimuth) (Schwarz and Wei 1999). Consequently, the transformation matrix  $\mathbf{R}_{\rm b}^{\ell}$  is determined as follows (Britting 1971):

$$\mathbf{R}_{b}^{\ell} = \begin{bmatrix} \left(\mathbf{g}^{\ell}\right)^{T} \\ \left(\mathbf{\omega}_{ie}^{\ell}\right)^{T} \\ \left(\mathbf{g}^{\ell} \times \mathbf{\omega}_{ie}^{\ell}\right)^{T} \end{bmatrix}^{-1} \begin{bmatrix} \left(\mathbf{f}^{b}\right)^{T} \\ \left(\mathbf{\omega}_{ie}^{b}\right)^{T} \\ \left(\mathbf{g}^{b} \times \mathbf{\omega}_{ie}^{b}\right)^{T} \end{bmatrix}, \tag{1}$$

where

f<sup>b</sup>: The vector of the accelerometers specific force measurements,

 $\omega_{i_e}^b$ : The vector of the gyroscopes angular velocity measurements,

 $\mathbf{g}^{\ell}$ : The vector of the Earth's gravity represented in the  $\ell$ -frame.

 $\mathbf{\omega}_{i_e}^{\ell}$ : The vector of Earth rotation rate represented in the  $\ell$ -frame.

The coarse alignment rapidly sets the  $\mathbf{R}_b^\ell$  matrix to an approximate value so that, during the fine alignment stage, first order approximations can be safely applied while modeling the residual misalignment errors (Savage 2000). The inertial sensor errors are usually modeled as random walk or 1st order Markov process (Gelb 1974). The estimation of these errors is usually based on the Kalman filtering algorithm (Brown and Hwang 1992), which employs the condition of zero velocity update (ZUPT) to provide optimal estimate of the misalignment errors (Schwarz and Wei 1999). Fine alignment brings the  $\mathbf{R}_b^\ell$  matrix to an accuracy sufficient to initiate the navigation mode.

## 1.2 Limitations of Present Methodologies

The estimation accuracy of the initial attitude angles is usually limited by the relatively low signal to noise ratio (SNR) of the gyroscope measurements, which may lead to inaccurate estimation of the azimuth angle (Noureldin et al. 2001). Moreover, this poor signal to noise ratio results in spending relatively long time (10–15 minutes) during the fine alignment stage to allow Kalman filtering algorithm to converge (Algrain and Ehlers 1995). Furthermore, if low cost and tactical grade inertial sensors are utilized, the high noise level at their measurements may jeopardize the accuracy of the overall alignment procedure (Abdel-Hamid et al. 2002).

### 1.3 Aim of the Present Article

This paper aims at:

(1) suggesting a new adaptive pre-filtering method to improve the SNR of inertial sensors;

(2) investigating the impact of this pre-filtering method on the estimation accuracy and the speed of convergence of the Kalman filtering algorithm during the alignment process.

## 2 Adaptive Pre-Filtering of Inertial Sensor Measurements

In order to reduce the noise level of the inertial sensor measurements, transversal tap delay finite impulse response (FIR) filters are utilized at their outputs. Each FIR filter consists of a set of a delay elements and tap weights as shown in Fig. 1 (Haykin 1996). The filter output is related to the input sequence through the filter tap weights as follows:

$$y(n) = \mathbf{W}^{T} \mathbf{U}(n) = \sum_{k=1}^{M} W_{k} \ u(n-k+1) ,$$
 (2)

where y(n) is the filter output,  $\mathbf{W} = [W_1 \ W_2 \ ... \ W_M]^T$  are the filter tap weights and  $\mathbf{U}(n) = [u(n) \ u(n-1) \ ... \ u(n-M+1)]^T$  is the vector of input sequence. The optimal values of the tap weights of each filter are independently determined using the adaptive filtering theory (Solo and Xuan 1995).

# 2.1 FIR Filter Design

As mentioned earlier, a separate FIR filter is designed at the output of each inertial sensor. The design of FIR filter tap weights using the adaptive filtering theory necessitates mounting the IMU at a perfectly known orientation. This known orientation is required only once (for example in laboratory environment) for the design of the filter. Once the filter is designed, its tap weights can be utilized at any other orientation to limit the noise level at the output of the inertial sensors. At this known orientation, the pitch, the roll and the azimuth should be precisely known. These attitude angles are utilized to determine the corresponding transformation matrix  $\mathbf{R}_{\mathrm{b}}^{\ell}$ . Consequently, the corresponding theoretical inertial sensor measurements can be computed as

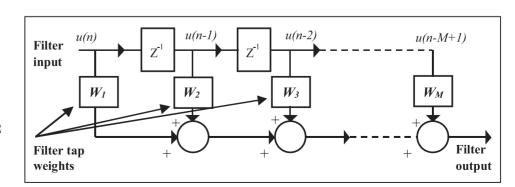


Fig. 1: Structure of a tap-delay FIR filter with *M* tap weights

$$\begin{bmatrix} f_{xo} \\ f_{yo} \\ f_{zo} \end{bmatrix} = \mathbf{R}_{\ell}^{b} \quad \mathbf{g}^{\ell} = \mathbf{R}_{\ell}^{b} \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} , \tag{3}$$

$$\begin{bmatrix} \boldsymbol{\omega}_{xo} \\ \boldsymbol{\omega}_{yo} \\ \boldsymbol{\omega}_{zo} \end{bmatrix} = \mathbf{R}_{\ell}^{b} \quad \boldsymbol{\omega}_{ie}^{\ell} = \mathbf{R}_{\ell}^{b} \begin{bmatrix} 0 \\ \boldsymbol{\omega}^{e} \cos \boldsymbol{\varphi} \\ \boldsymbol{\omega}^{e} \sin \boldsymbol{\varphi} \end{bmatrix}, \tag{4}$$

where  $f_{xo}$ ,  $f_{yo}$  and  $f_{zo}$  are the values of specific forces and  $\omega_{xo}$ ,  $\omega_{yo}$  and  $\omega_{zo}$  are the values of angular velocities corresponding to the IMU known orientation during the design process, g is the vertical gravity component that can be determined using the normal gravity model,  $\omega^e$  is the Earth rotation rate about its spin axis (15.04°/hr) and  $\varphi$  is the latitude. The above values of specific forces and angular velocities are utilized during the design process as the desired response of the corresponding inertial sensor measurement.

The FIR filter tap weights are, then, adapted by comparing the output of the filter y(n) to the desired response d(n) (see Fig. 2) (Haykin 1996). An adequate learning algorithm uses the estimation error e(n) between y(n) and d(n) to adapt the filter tap weights to their optimal values. The desired response d(n) is the anticipated errorfree signal at the output. Therefore, when the filter output y(n) is compared to d(n) the result is the error for which the FIR filter will be trained to remove. Fig. 2 shows a block diagram of the learning process.

## 2.2 FIR-Filter Training Procedure

The learning criterion used for the determination of the optimal values of the filter tap weights employs the minimization of the mean-square value of the estimation error e(n) (Haykin 1996):

$$J(n) = E \left[ e^2(n) \right], \tag{5}$$

where J(n) is the expectation of the square value of the estimation error, known as the mean-square estimation error (MSEE) or the cost function.

The estimation problem is related to selecting the filter tap weights vector  $\mathbf{W} = [W_1 \ W_2 \ \dots \ W_M]^T$  that minimizes the estimation errors in the mean square sense. Eq. (5) describes a quadratic surface with a minimum at  $W = W_0$ , where  $\mathbf{W}_0$  is the column vector containing the optimal values of the tap weights (Haykin 1996). Since this surface has a unique minimum, the optimal solution  $\mathbf{W}_{o}$  is also unique. Several techniques have been developed for the determination of the optimal tap weights and are known by the learning criterion: e.g. least mean square (LMS), recursive least square (RLS), and etc. (Solo and Xuan 1995). However, it has been reported that the LMS criterion is more adequate when the SNR of the input sequence  $\mathbf{U}(n)$  is relatively poor, which is the case with the gyroscope measurements (Solo and Xuan 1995). The LMS technique is used to design the FIR filter and determine adaptively the optimal values of the tap weights. According to the normalized LMS criterion, the tap weights are iteratively adjusted towards their optimal values using the following update equation (Solo and Xuan, 1995):

$$\begin{bmatrix} W_{1}(n+1) \\ W_{2}(n+1) \\ \vdots \\ W_{M}(n+1) \end{bmatrix} = \begin{bmatrix} W_{1}(n) \\ W_{2}(n) \\ \vdots \\ W_{M}(n) \end{bmatrix} + \frac{1}{\mathbf{U}^{T}(n)\mathbf{U}(n)} e^{(n)} \begin{bmatrix} u(n) \\ u(n-1) \\ \vdots \\ u(n+M-1) \end{bmatrix}.$$

$$(6)$$

The first sample of the filter output y(n) is obtained after collecting a number of input samples equal to the number of tap weights. The filter output is then compared to the desired response d(n), and the error between them is applied to the tap weights update equation until the

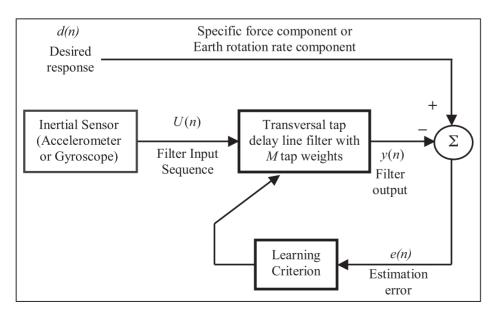


Fig. 2: Block diagram of the design process of the FIR filter

MSEE reached its steady-state value, and the filter tap weights reached their optimal values.

Therefore, this transversal filter results in a time delay at the output of each inertial sensor, which increases with the filter order.

## 2.3 Performance of the proposed FIR Filter

During the alignment process, the optimal tap weights are utilized to reduce the output noise of the corresponding inertial sensor. The values of these tap weights are then used to reduce the measurement noise for other alignment processes at different orientations. Since the desired components of Earth gravity or Earth rotation rate have extremely slow variations with time, the designed filters are basically low pass filters with small cutoff frequencies. The minimization of the inertial sensor output noise depends on the order of the filter. Higher filter orders provide lower cutoff frequencies, thus reducing the measurements noise and attenuating the amplitude of high frequency dynamics.

The real-time implementation of the filter, with its tap delay structure, necessitates collecting a number of samples from the input sequence equal to the number of tap weights (i. e. the filter order) before delivering the output.

# 3 Modified Alignment Procedure

The pre-filtering method helps in reducing the measurement noise of the inertial sensors, thus providing more accurate and faster coarse alignment. However, fine alignment procedures based on the Kalman filtering algorithm is still applied to improve the estimation accuracy.

The modified alignment procedure starts with averaging the inertial sensor measurements within certain time interval (usually less than one second). This averaging process helps in two aspects. First, it removes some of the high frequency noise components and reduces the uncertainty so that a better filter performance can be achieved. Second, the number of tap weights (i. e. the filter order) becomes significantly less than that when the averaging process is not performed. This averaging process results

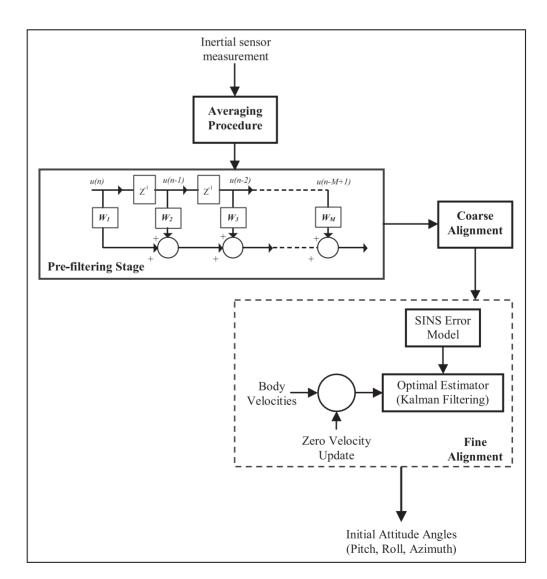


Fig. 3: Block diagram of the modified alignment procedure

in reducing the data rate at the output of the sensor. It should be highlighted that the FIR filter should be designed at the same data rate as the one used during the alignment procedure. After averaging, the measurement is then processed by the FIR filter tap weights to reduce their noise level before applying the coarse and fine alignment procedures. A block diagram of the modified alignment procedure is shown in Fig. 3. The proposed pre-filtering methodology helps Kalman filtering to converge faster during the fine alignment stage and to provide better estimate of the initial attitude angles.

#### 4 Results and Discussion

# 4.1 Pre-Filtering of Inertial Sensor Measurement

The proposed pre-filtering method has been applied to two different types of IMUs. The first IMU is LN-250, which incorporates three mutually orthogonal fiber-optic type gyroscopes and three-axis accelerometers. The second IMU is LTN-90-100, which utilizes ring laser gyroscopes. Both units are characterized as navigational grade IMUs with drift characteristics of less than 0.02°/hr. Tab. 1 gives a summary of the manufacturer specifications for LN-250 and LTN-90-100.

The averaging procedure was applied at 0.1 s time interval for the LN-250 and 0.125 s time interval for the LTN-90-100. This results in reducing the data rate to 10 Hz for the LN-250 and 8 Hz for the LTN-90-100. The pre-filtering stage was designed for both IMUs to only impose 100 s time delay at the output of each inertial sensor. Thus, 1000 and 800 tap weights were considered for the LN-250 and the LTN-90-100, respectively.

Fig. 4 presents the learning curve showing the MSEE and its convergence over time for the LN-250 gyroscope  $\omega_x$ . It is evident that the MSEE decreased progressively until reaching a steady state value. This steady state value of the MSEE has been achieved after about 15 seconds from the beginning of the design process. Once the learning curve converged to its steady state, the values of the tap weights could no longer change and the learning process was stopped, since no further improve-

Tab. 1: The manufacturer specifications for LN-250 and LTN-90-100

	LN-250	LTN-90-100	
Bias error	Gyro: 0.008 deg/hr Accel.: 40 μg	Gyro: 0.01 deg/hr Accel.: 50 μg	
Angle random walk	0.0015 deg/√hr	0.0025 deg/√hr	
Scale factor error	Gyro: 15 ppm Accel: 50 ppm	Gyro: 5 ppm Accel: 50 ppm	
Data rate	200 Hz	64 Hz	

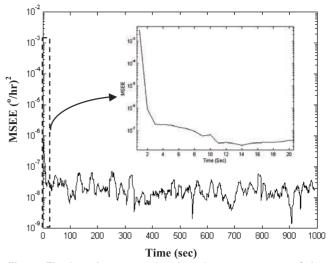


Fig. 4: The learning curve showing the convergence of the mean square estimation error (MSEE) to its minimal value during the design process

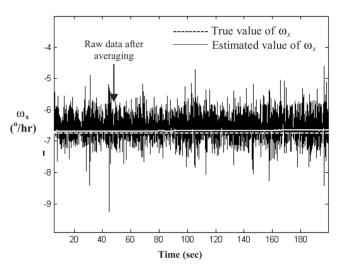


Fig. 5: Comparison of the raw data after averaging and after the pre-filtering stage with respect to the true value

ment to the values of the tap weights could be expected after the MSEE reached its steady state value.

After completing the design process, the values of the tap weighs were fixed and have been utilized to reduce the measurement noise of such inertial sensor. The cutoff frequencies of the six filters designed for the set of the LN-250 inertial sensors were between 0.001 and  $0.002 \,\mathrm{Hz}$ . The LN-250- $\omega_{\mathrm{r}}$  gyroscopic measurements have been processed by these tap weights at the pre-filtering stage (see Fig. 5). The averaging and pre-filtering procedures resulted in reducing the standard deviation of the measurement noise from 3.05°/hr to 0.48°/hr after averaging and to 0.013°/hr at the output of the FIR filter. As can be depicted from Fig. 5, the Earth rotation component monitored by  $\omega_r$  is now estimated precisely with an error of less than 0.07°/hr when compared to the reference value. Tab. 2 summarizes the effect of the proposed pre-filtering method on the different inertial sensors of LN-250 in terms of the estimation error of the Earth gravity and Earth rotation rate components as well as the

Inertial STD before STD after STD after **Estimation** Units Mean Sensor FIR adaptive any Averaging error processing at 0.1 sec. filtering interval -6.6983.0483 0.4805 0.013 0.0733 deg/hr  $\omega_{r}$ deg/hr 7.0062 0.0100 2.3105 0.7015 0.0072  $\omega_{\nu}$ deg/hr -11.52 3.5573 0.7534 0.0181 0.0067  $\omega_{2}$  $m/s^2$ 0.1156 0.0176 0.0016 2.0600e-5 8.3561e-5  $m/s^2$ 0.2727 0.0262 0.0023 4.1933e-4 1.3001e-4 -9.803 0.0185 0.0017 1.6620e-5 4.6520e-5  $m/s^2$ 

Tab. 2: Pre-filtering results for the LN-250 inertial sensors (FOG gyros)

Tab. 3: Pre-filtering results for the LTN-90-100 inertial sensors (Ring laser gyros)

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Inertial Sensor	Mean	STD before any processing	STD after Averaging at 0.125 sec. interval	STD after FIR adaptive filtering	Estimation error	Units
$\omega_{x}$	-9.576	174.7855	16.8826	0.0491	0.0855	deg/hr
$\overline{\omega_{y}}$	-1.075	168.0583	20.6500	0.0202	0.0043	deg/hr
$\omega_z$	11.872	197.8474	18.9625	0.0430	0.1369	deg/hr
$\overline{f_x}$	-0.081	0.0253	0.0039	1.4899e-5	0.0022	m/s <sup>2</sup>
$\overline{f_y}$	-0.089	0.0311	0.0051	1.8646e-5	2.1537e-4	m/s <sup>2</sup>
$f_z$	9.807	0.0326	0.0042	8.706e-6	3.888e-4	m/s <sup>2</sup>

corresponding noise level. It should be highlighted that the success of the proposed method necessitates accurate computation of the biases of the different inertial sensors. The bias errors of inertial sensors might have been contributed to the estimation errors shown on Tab. 2. It can be also depicted from this table that the estimation error for the accelerometer  $f_y$  (1.3001 · 10<sup>-4</sup> m/s²) is higher than the other two accelerometers (8.3561 · 10<sup>-5</sup> m/s² and 4.6520 · 10<sup>-5</sup> m/s²). This is basically due to the relatively higher noise level of the measurements at the output of this accelerometer (check column 3 of the same table). In order to achieve the same level of estimation error, the accelerometer  $f_y$  should be processed by higher FIR filter order (i. e. larger number of tap weights).

The same pre-filtering method was applied to the LTN-90-100. The standard deviation of the noise associated with the gyroscopes raw measurements of this unit is higher than that of the LN-250. This is basically due to the different technologies used for the gyroscopes. Tab. 3 lists the pre-filtering results for the LTN-90-100 and its impact in reducing the noise level at the output of the inertial sensors.

#### 4.2 Estimation of the tilt and heading angles

The pre-filtering stage was capable of delivering the inertial sensor measurements with less amount of noise, thus

providing more precise coarse alignment. This can be shown for the azimuth angle on Fig. 6 for the LN-250 and on Fig. 7 for the LTN-90-100. The fine alignment procedure utilized Kalman filtering algorithm to estimate the attitude angles. Fig. 6 shows the azimuth angle during the fine alignment procedure with and without the prefiltering stage for the LN-250 data. It is obvious that, after applying the pre-filtering stage, the azimuth converged in about 150 s faster than the other solution (without prefiltering), which takes about 500s to converge. The difference between the two solutions (i. e. with and without pre-filtering) at the end of the fine alignment procedure was less than 0.1° for the azimuth, 0.002° for the pitch and 0.013° for the roll. The relatively higher error of the roll angle if compared to the pitch angle is basically due to the higher estimation error at the output of the pre-filtering stage for the  $f_{\nu}$  accelerometer (see Tab. 2).

The same fine alignment procedure was applied to the LTN-90-100 data with and without pre-filtering. The result for the azimuth is shown on Fig 7. In addition to the faster convergence (about 250 seconds compared to 600 seconds for the case without pre-filtering), the difference between the two estimates is less than 0.1°.

The inaccurate estimation of the inertial sensor biases contributed to above errors. It can be observed from Fig. 5 that the estimate of the Earth's rotation rate component monitored along the sensitive axis of the gyro slightly deviates from the reference value by approxi-

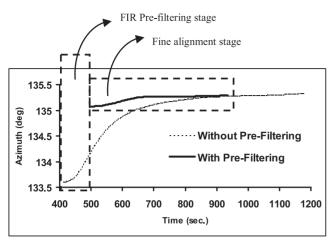


Fig. 6: The azimuth angle during fine alignment procedure with and without the pre-filtering stage for the LTN-250 data

mately 0.07°/hr. This is most likely due to bias errors. As mentioned earlier, accurate determination of the biases by an appropriate calibration procedure is necessary for the successful implementation of the proposed modified alignment procedure. However, it should be highlighted here that the error between the two solutions might be due to inappropriate estimation of the attitude angles when the pre-filtering stage is not applied, which corresponds to high measurement uncertainty as can be determined from Tab. 1 and 2 for different gyros and accelerometers.

## 5 Conclusions

Most of the present alignment procedures suffer from the relatively high noise levels of the inertial sensors measurements. This leads to relatively long time for inertial system alignment and may lead to inaccurate estimation of the initial attitude angles. This article suggested a modified alignment procedure, which utilized an FIR prefiltering stage to limit the noise level at the output of inertial sensor before starting the conventional alignment procedures. The results showed that the modified alignment procedure could be performed in less than 300 seconds to give accurate estimation of the initial attitude.

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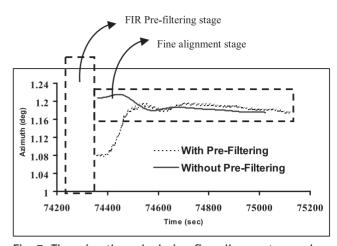


Fig. 7: The azimuth angle during fine alignment procedure with and without the pre-filtering stage for the LTN-90-100 data

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