The Impact of Correcting Measurements of Laserscanners on the Uncertainty of Derived Results

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Summary

Laserscanners are applied to determine the geometry of manufactured objects. The geometry often follows from functions of estimates computed from measured coordinates of points. The uncertainty of the derived results is additionally needed. The coordinates are obtained by distances, which have to be corrected due to systematic effects. The impact of the corrections on the uncertainty of the derived results is investigated. In this paper the geometry of the section of two planes is determined. The uncertainty of the results is obtained by Monte Carlo simulations which provide standard deviations, covariances and also confidence limits. It turns out that the correlations introduced by correcting the measurements should be considered.

Zusammenfassung

Laserscanner werden eingesetzt, um die Geometrie gefertigter Objekte zu bestimmen. Die Geometrie folgt häuf ig aus Funktionen von Schätzungen, die aus den gemessenen Koordinaten von Punkten berechnet werden. Die Unsicherheit der abgeleiteten Ergebnisse wird zusätzlich benötigt. Die Koordinaten ergeben sich aus Entfernungen, die wegen systematischer Effekte korrigiert werden müssen. Die Wirkung dieser Korrekturen auf die Unsicherheit der abgeleiteten Ergebnisse wird untersucht. Bei der hier vorliegenden Aufgabe ist die Geometrie des Schnitts zweier Ebenen zu bestimmen. Die Unsicherheit der Ergebnisse wird durch Monte-Carlo-Simulationen abgeleitet, die die Standardabweichungen, die Kovarianzen und ebenfalls die Konf idenzgrenzen ergeben. Es stellt sich heraus, dass die durch Korrektionen der Messungen eingeführten Korrelationen berücksichtigt werden sollten.

1 Introduction

The coordinates of points on the surface of manufactured objects are measured by laserscanners to obtain the geometry of objects. The uncertainty of results derived from the measurements like functions of estimates are also needed. Variances and covariances of observations can be determined by repeated measurements, cf. Koch (1999, p. 250). However, these results contain only a part of the uncertainty. Adding corrections due to systematic effects of the measurements will change the uncertainty of the derived results. Within the scope of this paper these changes will be investigated.

The accuracy of measurements will be defined as uncertainty according to the »Guide to the Expression of Uncertainty of Measurements (GUM)«, ISO (1995), which has been internationally accepted as a standard for evaluating uncertainties in measurements, cf. Sommer and Siebert (2004). GUM groups the components of uncertainty according to the methods of determination into Type A and Type B. Type A is obtained by statistical methods, while Type B lists experience and knowledge about the measuring instrument. The uncertainty of Type A as well as of Type B is given by variances and covariances. Both types may contain random and systematic effects of the measurements. Consequently, the systematic effects in measurements are defined as random variables.

Although not explicitly stated, GUM applies Bayesian statistics for evaluating Type B uncertainty, cf. Weise and Wöger (1993). Unknown quantities or parameters are introduced in the Bayesian approach as random variables, cf. Koch (2007, p. 34), contrary to traditional statistics with fixed quantities. It is useful to regard the systematic effects as random variables, because in Bayesian statistics not the systematic term is random but the knowledge about its value.

The uncertainty of measurements is expressed by variances and covariances which are obtained for derived results by the law of error propagation, cf. Koch (1999, p. 100). However, Monte Carlo simulations may also be applied to compute variances and covariances so that the error propagation can be avoided. GUM recommends computing the expanded uncertainty of a measurement which is expressed by Bayesian confidence interval. It can also be obtained by Monte Carlo simulations, even if the measurements are not normally distributed, Koch (2008b).

Measurements of a laserscanner for determining the geometry of an object are analyzed. The geometry follows from fitting three-dimensional planes to measurements. Examples for fits of free-form surfaces are given by Koch (2009). The planes are fitted to the three coordinates measured by laserscanner so that three measurements appear in each observation equation. The linear model can therefore be only applied, if additional unknown parameters are introduced, cf. Koch (2007, p. 88). This, however, would considerably increase the number of unknown parameters. Alternatively the method of total least squares could be considered, cf. Golub and Loan (1980), Schaffrin and Wieser (2008) and also orthogonal distance regression Forbes (2006). However, both methods ask for special structures of the covariance matrix of the observations. When determining uncertainty, the covariance matrix of the coordinates measured by the laserscanner should be considered, cf. Koch (2008a). The nonlinear Gauß -Helmert model is therefore applied. Of course, linearization is critical, the unknown

parameters as well as the observations need approximate values, see Koch (2002), Neitzel and Petrovic (2008). The linearization has to be applied iteratively. Nevertheless, accurate approximate values can be provided so that after one or two iterations convergence is reached.

Effects causing errors of measurements of laserscanners have been examined. Schulz and Ingensand (2004) investigate errors resulting from deviations from the perpendicular rotation axis of the instrument and determine corrections of the measured distances and corrections due to incidence angles and reflecting material. Böhler (2005) investigates the reflectivity of surfaces, the effects of targets, the resolution at object and distance depending effects. Reshetyuk (2006) sets up a test field consisting of points with known coordinates to test three laserscanners. Gräfe (2007) calibrates a laserscanner for distance depending effects, effects of varying incidence angles and the reflectivity of surfaces being scanned. To obtain comparable results, a standardized calibration using test objects has been suggested by Heister (2006). These calibrations do not consider the correlations of measurements of the laserscanners and the correlations caused by correcting the measurements when determining the uncertainty of the derived results.

A system calibration for that specific task at hand which considers the measuring system as a whole would be desirable. A first step in this direction is the determination of the variances and covariances of the measurements which are taken by the laserscanner for the special application. These variances and covariances are computed from repeated observations and are independent from systematic effects of the observations, Koch (2008a). Repetitions are, nevertheless, needed and they are feasible because of the rapid data acquisition by laserscanners. Effects distorting the measurements have to be independently determined. This approach is taken here.

An addition constant of laserscanner Leica HDS 3000 of the Institute for Geodesy and Geoinformation (IGG) and a Type B component of uncertainty are considered. Distance depending effects and effects of incidence angles are determined on an interferometer-based calibration line of the IGG, described by Witte and Yang (1994). The lengths of the calibration line are determined by differences of measurements of the laserscanner to eliminate the influence of addition constant. When analyzing these observations two measurements appear in an observation equation so that the linear model cannot be applied like for fitting planes mentioned above. However, only one additional unknown parameter needs to be introduced to use the linear model.

The fit of planes using the Gauß-Helmert model is derived in sec. 2. Sec. 3 shows the analysis of distances of the laserscanner taken on the calibration line. The covariance matrix of the distances corrected by systematic effects is discussed in sec. 4, while sec. 5 maps the analysis of observations of the laserscanner. The task to be solved is the determination of uncertainty of the geometry of the

section of two planes of an object constructed for the investigation.

2 Fitting Three-Dimensional Planes

Let x, y and z be the coordinates of the rectangular coordinate system of the laserscanner with the origin at the center of instrument. The x axis lies with a good approximation horizontally, the z axis points to the zenith and the y axis coincides with the center of lines of sight of the laserscanner. Let x_i, y_i, z_i with $i \in \{1, \dots, n_p\}$ be the coordinates of a grid of n_p points lying on a three-dimensional plane and be measured by the laserscanner. The grid is established by the x_i and z_i coordinates. Let β_0, β_1 and β_2 be the unknown parameters of the three-dimensional plane. In order to estimate them the observation equations of the nonlinear Gauß-Helmert model are introduced

$$
\begin{aligned}
f_i \;=\; & \beta_0 + \beta_1(x_i + e_{x_i}) + \beta_2(z_i + e_{z_i}) - y_i - e_{y_i} = 0 \\
& \text{for} \quad i \in \{1, \dots, n_p\}
\end{aligned} \tag{1}
$$

where f_i denotes the function depending on the unknown parameters and the errors $e_{x_i}, e_{y_i}, e_{z_i}$ of the measured coordinates. The $3n_p \times 3n_p$ positive definite covariance matrix Σ of the vector y of measurements is given by

$$
D(y) = \Sigma \tag{2}
$$

with

$$
y = |x_1, y_1, z_1, \dots, x_{n_p}, y_{n_p}, z_{n_p}| \,. \tag{3}
$$

As already mentioned in the introduction, approximate values β_{00}, β_{10} and β_{20} have to be introduced for the 3×1 vector of unknown parameters $\beta = (\beta_k)$ with $k \in \{0, 1, 2\}$ and for the measurements x_i, y_i, z_i to linearize (1), see Koch (2002). With the $n_p \times 3$ matrix X, the $n_p \times 3n_p$ matrix Z, the $n_p \times 1$ vector w

$$
\begin{aligned}
X \;=\; & \left(\left.\frac{\partial f_i}{\partial \beta_k}\right|_0\right), \; Z = \left(\left.\frac{\partial f_i}{\partial x_i}\right|_0, \left.\frac{\partial f_i}{\partial y_i}\right|_0, \left.\frac{\partial f_i}{\partial z_i}\right|_0\right) \\
w \;=\; & (\beta_{00} + \beta_{10}x_i + \beta_{20}z_i - y_i)
\end{aligned} \tag{4}
$$

and the $n_p \times 1$ vector k of Lagrange multipliers, the normal equations for the estimate $\hat{\beta}$ of β are obtained

$$
\left|
\begin{matrix}
Z\Sigma Z' & X \\
X' & 0
\end{matrix}
\right|
\left|
\begin{matrix}
k \\
\hat{\beta}
\end{matrix}
\right|
=
\left|
\begin{matrix}
-w \\
0
\end{matrix}
\right| \,. \tag{5}
$$

The solution follows with

$$
\begin{aligned}
\hat{\beta} \;=\; & -(X'(Z\Sigma Z')^{-1}X)^{-1}X'(Z\Sigma Z')^{-1}w \\
k \;=\; & (Z\Sigma Z')^{-1}(-w - X\hat{\beta}) \\
\hat{e} \;=\; & \Sigma Z'k
\end{aligned} \tag{6}
$$

where $\hat{e} = (|\hat{e}_{x_i}, \hat{e}_{y_i}, \hat{e}_{z_i}|')$ denotes the vector of residuals.

By applying (6) iteratively with recomputing (4) for each step of iteration the approximate values are improved. As a first approximation for coordinates the measured values can be taken because of their small variances for application in sec. 5. The approximate values β_{00}, β_{10} and β_{20} for the unknown parameters follow from the fit of plane in the linear model

$$\beta_{00} + \beta_{10}x_i + \beta_{20}z_i - y_i - e_{y_i} = 0 \ . \qquad (7)$$

This adjustment gives good approximations, due to the fact that the standard deviations of x_i and z_i are about a factor of 10 smaller than the ones of y_i, as will be shown in sec. 5.

Let $\beta_{01}, \beta_{11}, \beta_{21}$ be the parameters of a first plane and $\beta_{02}, \beta_{12}, \beta_{22}$ the parameters of a second one, a point with coordinates x_s, y_s, z_s of the section of the two planes is then determined by

$$\beta_{01} + \beta_{11}x_s + \beta_{21}z_s = \beta_{02} + \beta_{12}x_s + \beta_{22}z_s = y_s \ , \quad (8)$$

thus

$$
\begin{aligned}
x_s &= \frac{1}{\beta_{11} - \beta_{12}}(\beta_{02} - \beta_{01} + (\beta_{22} - \beta_{21})z_s) \\
y_s &= \beta_{01} + \beta_{11}x_s + \beta_{21}z_s
\end{aligned}
\qquad (9)
$$

so that given z_s the coordinates x_s and y_s of a point of the section are computed by (9).

Incidence angles of the line of sight of the laserscanner with respect to the normal of a point on plane have to be computed for the following. Let $x_1 = |x_1, y_1, z_1|'$ and x_2, x_3 accordingly be points on the plane. The normal $n = |n_x, n_y, n_z|'$ to the plane at point x_1 is given by the vector product

$$n = (x_2 - x_1) \times (x_3 - x_1) \ . \qquad (10)$$

The incidence angle γ of the line of sight defined by x_1 with respect to the normal n follows from the inner product

$$\cos\gamma = \frac{x_1 x_n + y_1 y_n + z_1 z_n}{(x_1^2 + y_1^2 + z_1^2)^{1/2}(x_n^2 + y_n^2 + z_n^2)^{1/2}} \ . \qquad (11)$$

3 Measurements on the Calibration Line

Different lengths of the calibration line are realized by a sled which is moved on straight rails. The sled carries a prism for the interferometer and above it the target for the laserscanner with an inner circle of a diameter of 5 cm with a white surface and an outer circle of a diameter of 15 cm with a blue color. The target can be rotated around a vertical axis so that incidence angles between 0^{gon} and 100^{gon} may be chosen. The scans of target therefore have to be arranged symmetrically with respect to the center of target. This is achieved by a specific automatic measuring process which scans very densely first the whole target
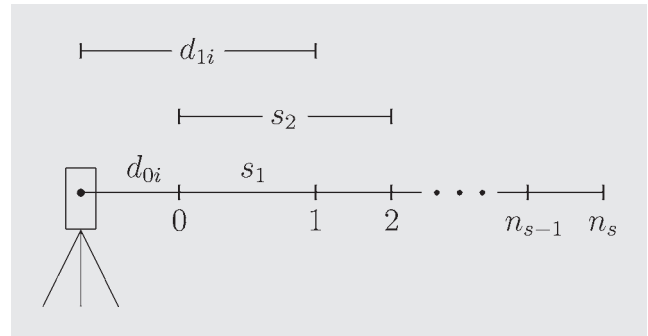

Fig. 1: Measurements of the laserscanner on the calibration line

and then the inner white circle to give the coordinates of center, cf. Kern (2003, p. 110). The interferometer is positioned at the end of calibration line and determines the lengths of the line by differences between the start of the line and the selected positions of the sled with the targets. The laserscanner stands at the start of the line and measures also differences between the position of the target at the start of the line and the chosen positions for the sled, see Fig. 1. Addition constants of the interferometer and the laserscanner cancel by this setup of measurements.

Let s_j with $j \in \{1, \ldots, n_s\}$ be the distances measured from differences by the interferometer with n_s being the number of distances and the number of positions of the sled. Their standard deviations are much smaller than the ones of the laserscanner so that their errors can be neglected. Let d_{0i} with $i \in \{1, \ldots, n_b\}$ be the distances measured by the laserscanner to the beginning of the calibration line and $d_{1i}, d_{2i}, \ldots, d_{n_s i}$ the distances to the n_s positions of the sled on the calibration line with n_b being the number of repeated measurements, see Fig. 1. The distances are derived from the three coordinates measured by the laserscanner.

The correction of the distance of the laserscanner depending on the distances s_j determined by the interferometer is denoted by α_1 so that the observation equation for estimating α_1 follows with

$$
\begin{aligned}
\alpha_1 s_j &= s_j - ((d_{ji} + e_{ji}) - (d_{0i} + e_{0i})) \\
&\quad \text{with} \quad i \in \{1, \ldots, n_b\} \\
&\quad \text{and} \quad j \in \{1, \ldots, n_s\}
\end{aligned}
\qquad (12)
$$

where e_{0i} and e_{ji} are errors of the distances of the laserscanner. Two observations are contained in one observation equation which does not lead to a linear model. However, by replacing the second observation plus error by the unknown parameter α_0 the observation equations of a linear model are obtained

$$
\begin{aligned}
\alpha_0 &= d_{0i} + e_{0i} \\
\alpha_1 s_1 - \alpha_0 &= s_1 - d_{1i} - e_{1i} \\
\alpha_1 s_2 - \alpha_0 &= s_2 - d_{2i} - e_{2i} \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\alpha_1 s_{n_s} - \alpha_0 &= s_{n_s} - d_{n_s i} - e_{n_s i} \ .
\end{aligned}
\qquad (13)
$$

The intensity of the reflected signal of a laserscanner reduces with $\cos\gamma$ where γ denotes the incidence angle of the line of sight with respect to the normal of the reflecting surface. The correction due to an incidence angle therefore follows with $\cos(100^{\mathrm{gon}} - \gamma) = \sin\gamma$. Let α_2 be the correction to be estimated for the incidence angles γ_k with $k \in \{1, \ldots, n_a\}$ and $\gamma_k \neq 0$. Let d_{jik} with $i \in \{1, \ldots, n_b\}$, $j \in \{1, 3, \ldots, n_s\}$, if every second point is selected for an odd number n_s of positions, be the distances measured by the laserscanner to the target which is rotated to get n_a different incidence angles. Let e_{jik} be the errors of d_{jik}. The correction α_2 is estimated together with α_0 and α_1. The observation equations (13) are therefore augmented by the observation equations

$$
\begin{aligned}
\alpha_1 s_1 - \alpha_0 + \alpha_2 \sin\gamma_k &= s_1 - d_{1ik} - e_{1ik} \\
\alpha_1 s_3 - \alpha_0 + \alpha_2 \sin\gamma_k &= s_3 - d_{3ik} - e_{3ik} \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\alpha_1 s_{n_s} - \alpha_0 + \alpha_2 \sin\gamma_k &= s_{n_s} - d_{n_s ik} - e_{n_s ik} \,.
\end{aligned}
\tag{14}
$$

A least squares adjustment based on (13) and (14) then gives the estimates $\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2$ of $\alpha_0, \alpha_1, \alpha_2$. The inverse of the matrix of normal equations times the estimated variance factor yields the covariance matrix of the unknown parameters. The 2×2 covariance matrix of α_1, α_2 is only needed in the following. It is denoted by

$$
D\left(\left|\begin{array}{c} \alpha_1 \\ \alpha_2 \end{array}\right|\right) = \boldsymbol{\Sigma}_\alpha \,.
\tag{15}
$$

4 Covariance Matrix of Corrections

The data of the laserscanner are the x, y, z coordinates which are mainly affected by the measured distances so that corrections of the distances have to be determined. The geometry of the section of two planes, to be investigated here, is expressed in the coordinate system of the laserscanner. Thus, corrections of the z coordinates due to an error of the height index, cf. Kuhlmann et al. (2006), or corrections of the x coordinates do not need to be considered.

The corrections applied to the distances of laserscanner Leica HDS 3000 are the addition constant and the corrections depending on distances and incidence angles. The addition constant c has been estimated by Koch (2008a) with

$$
\hat{c} = -1.6 \,\text{mm} \quad \text{with} \quad \sigma_c = 0.7 \,\text{mm}
\tag{16}
$$

where σ_c denotes the standard deviation of c.

Distance depending corrections and corrections due to different incidence angles are estimated by the observation equations (13) and (14). Five distances s_j of 4 m, 6 m, 8 m, 10 m and 12 m are measured, thus $n_s = 5$ in (13) and (14). The distance measurements d_{ji} of (12) with $d_{0i} \approx 2.34$ m are repeated 5 times, therefore $n_b = 5$. For the lengths s_j of 4 m, 8m and 12 m the distances d_{jik} are determined with the incidence angles γ_k

of $33.33^{\mathrm{gon}}, 50.00^{\mathrm{gon}}, 66.67^{\mathrm{gon}}$ so that $n_a = 3$. The estimates $\hat{\alpha}_1, \hat{\alpha}_2$ of α_1, α_2 follow as

$$
\hat{\alpha}_1 = 0.146 \,\text{mm/m} \quad \text{and} \quad \hat{\alpha}_2 = 0.374 \,\text{mm} \,.
\tag{17}
$$

A measured distance of 10 m of the laserscanner therefore needs to be corrected by 1.46 mm and a line of sight with an incidence angle of $\gamma = 66.67^{\mathrm{gon}}$ by 0.32 mm. The corrections have to be projected onto the x, y, z axes to obtain the corrections for measured coordinates.

The correlation following from the 2×2 covariance matrix $\boldsymbol{\Sigma}_\alpha$ of (15) is 0.75, which means that the results for α_1 and α_2 are highly correlated. The confidence interval for α_1 computed by the t-distribution, cf. Koch (2007, p. 113), does not include the value zero. Thus, α_1 is significantly different from zero, but not α_2. Nevertheless, both corrections with their covariance matrix are applied because of the high correlation. The standard deviation of the distance measurements on the calibration line is estimated to be 0.82 mm. This result reflects the accuracy of the repeated scans of the target explained in sec. 3. It does not give the standard deviation of one measurement which lies between 2 mm and 3 mm according to the results for the y coordinates cited in sec. 5 which approximate the distances. The covariance matrix $\boldsymbol{\Sigma}_\alpha$ from (15) for α_1 and α_2 is therefore scaled by a factor of 2.5^2 to obtain a standard deviation of 2.0 mm for one distance. The scaled covariance matrix with unit m^2 is given by

$$
\boldsymbol{\Sigma}_\alpha = \left|\begin{array}{cc} 7.58 \times 10^{-9} & 6.54 \times 10^{-8} \\ & 9.99 \times 10^{-7} \end{array}\right| \,.
\tag{18}
$$

Not all systematic effects of the measurements of the laserscanner will be covered by the addition constant and the effect of distances and incidence angles. A Type B component of uncertainty is therefore assumed and introduced as individual correction for each coordinate with zero expectation and a standard deviation of 0.1 mm for the x and z coordinate and 1.0 mm for the y coordinate. These values are about half of the ones estimated for the standard deviations of the coordinates from repetitions as explained in sec. 5. This uncertainty increases the variances of the coordinates but does not change the covariances, as shown by Koch (2008a).

Let $\boldsymbol{s}$ be an $n_p \times 1$ vector of distances to the n_p points of the grid on the surface of a plane mentioned in sec. 2, let $\boldsymbol{a}$ be an $n_p \times 1$ vector of sines of the incidence angles and $\boldsymbol{1}$ an $n_p \times 1$ vector of ones, the $n_p \times 1$ vector $\boldsymbol{s}_c$ of corrections of the distances $\boldsymbol{s}$ is then determined with (16) and (17) by

$$
\boldsymbol{s}_c = \boldsymbol{1}\hat{c} + |\boldsymbol{s}, \boldsymbol{a}| \left|\begin{array}{c} \hat{\alpha}_1 \\ \hat{\alpha}_2 \end{array}\right| \,.
\tag{19}
$$

The estimates $\hat{c}$ and $\hat{\alpha}_1, \hat{\alpha}_2$ are independent and their variances and covariances follow from (16) and (18). The $n_p \times n_p$ covariance matrix $\boldsymbol{\Sigma}_{s_c}$ of $\boldsymbol{s}_c$ is therefore obtained by the law of error propagation with

$$
\boldsymbol{\Sigma}_{s_c} = \sigma_c^2 \boldsymbol{1}\boldsymbol{1}' + |\boldsymbol{s}, \boldsymbol{a}|\boldsymbol{\Sigma}_\alpha \left|\begin{array}{c} \boldsymbol{s}' \\ \boldsymbol{a}' \end{array}\right| \,.
\tag{20}
$$

All elements of the covariance matrix of the first product on the right-hand side of (20) are different from zero as well as all elements of the second product. Thus, correlations are introduced if distances are corrected by an addition constant or by effects depending on distances and incidence angles or by a combination of both corrections.

5 Determining the Geometry and its Uncertainty of the Section of Two Planes

The part of surface of the manufactured object which is determined by the laserscanner Leica HDS 3000 consists of two vertical, well reflecting planes which meet under a right angle in a sharp edge, see Fig. 2. The geometry of edge which cannot be scanned has to be determined together with its uncertainty. The position chosen for the instrument lies closer to the right plane than to the left plane. The distances to the left plane vary between 10.4 m and 9.2 m and to the right plane between 9.1 m and 9.6 m. The incidence angles of the left plane lie between 64^{gon} and 69^{gon} and of the right plane between 36^{gon} and 41^{gon}. The differences between the x and z coordinates of the points of the grid are about 0.13 m.

The expected values of the $3 \times 6 \times 6 = 108$ coordinates of the $6 \times 6 = 36$ points of the grid for each plane and the 108×108 covariance matrix are estimated from 600 repetitions of the scans in a special multivariate model, cf. Koch (2008a). The standard deviations of the x and z coordinates of the left plane lie below 0.3 mm and the standard deviations of the y coordinates vary between 1.9 mm and 3.1 mm. For the right plane these standard deviations are slightly smaller. The correlation matrix resulting from the covariance matrix contains absolute values of correlations up to 0.998 for the three coordinates of the grid points for the left plane. Changes in the measured y coordinates cause changes with the same sign, as for instance, in the x coordinates. This explains such high correlations. Above these correlations in the correlation matrix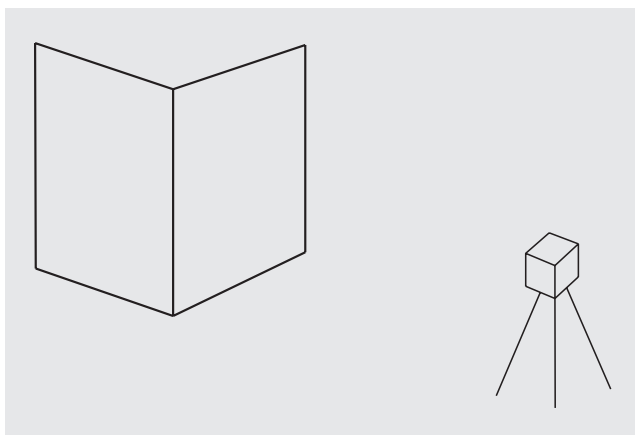, there are 4583 absolute values of correlations smaller than 0.1, 804 values between 0.1 and 0.2, 173 between 0.2 and 0.3 and 110 values between 0.3 and 0.79 which is the maximum value. For the right plane, the absolute values of the correlations are smaller, i. e. up to 0.987 for the three coordinates of the grid points and for the remaining correlations 4570 values smaller than 0.1, 1026 values between 0.1 and 0.2, 55 between 0.2 and 0.3 and 19 between 0.3 and 0.42 which is the maximum value.

To express the geometry of the section of two planes, three quantities are considered. The first one is the length of the section between two points computed by (9) for the z coordinates of the highest and lowest point of the vertical grid line to the right of the section. The second quantity is the distance by which the upper point of the section deviates from the z axis of the lower point. It gives orientation of the section with respect to the z axis. The third quantity consists of the sum of distances from the center of instrument to six points of the section. The z coordinates of these points are given by the vertical grid line to the right of the section.

As already mentioned in the introduction, Monte Carlo simulations are applied to determine variances and covariances of the results derived from the measurements and the corrected measurements. In addition, this avoids error propagation and gives confidence limits for the derived results. It is assumed that the measurements of the laserscanner are normally distributed with expected values and covariance matrix determined from the 600 repeated measurements mentioned above. A triangular distribution with its spiked shape models a correction very well, because the probability mass is concentrated in a small interval for a small standard deviation, so that a constant is simulated. The addition constant has been independently determined from the two corrections for distances and incidence angles. The addition constant therefore gets univariate triangular distribution with expectation and standard deviation from (16), while the latter two corrections get multivariate triangular distribution defined by Koch (2008a) with expected values from (17) and covariance matrix from (18). A univariate uniform distribution with zero expectation and variances given in sec. 4 is assumed for the Type B component of uncertainty.

Random values are generated for the coordinates of the points of grid. Two planes are fitted to the random variates by (6). When generating for the first time, the approximate values for the unknown parameters of planes are taken from the fit by (7) of the first scan of points of each plane. The estimated parameters for the first set of random variates serve as approximate values for the second one, the second one for the third and so on. This procedure gives seven significant figures for the estimated parameters when iteratively estimating twice for the first set of generated random values and once for the following times. The section of the two planes follows from (8) and the length of the section, its deviation from the

Fig. 2: Laserscanner and the two intersecting planes

Tab. 1: Expectations, standard deviations, 0.95 confidence intervals for three distances by Monte Carlo simulations

Geometric quantity	Expect.	Std. Dev.	Confidence Interval	
Measurements	[m]	[mm]	lower limit [mm]	upper limit [mm]
1. Length of the section				
without corrections	0.7916	0.27	−0.54	0.53
with corrections	0.7916	0.30	−0.59	0.59
with corrections & Type B comp.	0.7916	0.33	−0.65	0.65
2. Deviation of the section from the z axis				
without corrections	0.0100	0.64	−1.24	1.26
with corrections	0.0100	0.64	−1.24	1.25
with corrections & Type B comp.	0.0100	0.73	−1.42	1.45
3. Sum of distances to six points of the section				
without corrections	54.2770	4.00	−7.84	7.84
with corrections	54.2767	7.84	−15.30	15.22
with corrections & Type B comp.	54.2767	8.00	−15.63	15.60

z axis and its sum of the distance to six points of the section from (9). To obtain the confidence intervals for these three distances, the random variates for the coordinates of the points of the two planes are generated 100 000 times. This number is sufficiently high to determine the confidence limits with at least two significant figures, see Koch (2008b).

Tab. 1 shows the results of computations for each of the three distances describing the geometry of the section. The expected values are subtracted from the confidence limits for a simpler comparison. The first line for each of the three distances shows the result of measurements without corrections. To compute it, random variates are generated from multivariate normal distribution. The second line gives the result of the corrected measurements. For this result, random values from univariate and multivariate triangular distribution are generated. These random variates refer to distances. They are projected onto the x, y, z coordinates and added to the random values of multivariate normal distribution. The third line, finally, shows the result of measurements additionally corrected for the Type B component of uncertainty. It follows by adding the random variates from the uniform distribution.

The results of Tab. 1 show that the length of section of the two planes and its orientation in the coordinate system of the laserscanner are very accurately determined with standard deviations less than 1 mm. When adding the corrections of the measured distances due to the addition constant and the dependence on distances and incidence angles as well as adding the Type B component of uncertainty, the standard deviations and the confidence intervals hardly change. This is caused by the fit of each of the two planes to 36 points. The expected values remain identical for the first two distances and almost the same for the third one after adding the corrections. This can be explained by the compensation of the negative addition constant with the positive corrections depending on distances and incidence angles.

The results for the sum of distances to six points of the section behave differently (Tab. 1). By adding corrections to the measurements their covariances increase as shown by (20). For the left plane, there are 879 absolute values of correlations below 0.1, 607 values between 0.1 and 0.2, 1656 between 0.2 and 0.3 and 2528 between 0.3 and 0.80 which is the maximum value. For the right plane, there are 665 absolute values below 0.1, 653 between 0.1 and 0.2, 1619 between 0.2 and 0.3 and 2733 between 0.3 and 0.55 which is the maximum value. Although the distances from the instrument are measured to well determined points of the section of the two planes, the increase of correlations of the distances lets the standard deviation and the limits of confidence interval of the sum jump by a factor of about two. Adding the Type B component of uncertainty increases the variances without changing the covariances as mentioned in sec. 4. The correlations therefore decrease so that the standard deviation of the sum of the six distances increases by less than 0.2 mm.

6   Conclusions

The effect of corrections to the measurements of a laserscanner on the uncertainty of derived quantities has been investigated. It turns out that the effect is small if the derived result follows from a large number of measurements

like the fit of planes in this study. However, if the results are strongly influenced by corrected measurements like the sum of distances to the points of the section of two planes, adding corrections considerably changes the uncertainty. The reason is the increase of correlations due to corrections so that the correlations should be taken into account when computing the uncertainty of derived results.

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