A Global Height System – Following Heinrich Bruns (1878)

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Summary

In 1878 the astronomer Heinrich Bruns published his famous article "Die Figur der Erde" (The Figure of the Earth), where he proposed to represent the Earth's geometry by a global polyhedron. Thereby gravity potential at all polyhedron points provides the necessary height information. Making use of all measurement techniques of his time, Bruns showed that the realization of such a concept is possible, in theory. The practical implementation suffered, however, from the effect of tropospheric refraction on zenith angles and from the inaccessibility of the world's oceans to geodetic measurements.

Space techniques have revolutionized geodesy. Meanwhile, under the umbrella of the International Earth Rotation and Reference Systems Service (IERS), the International Association of Geodesy (IAG) provides a terrestrial reference system from a combination of the space techniques Very Long Baseline Interferometry, Satellite Laser Ranging, Global Navigation Satellite Systems and Doppler Orbitography and Radiopositioning. Its realization, the International Terrestrial Reference Frame (ITRF), published in intervals of typically five years, consists of a long list of precise station coordinates and velocities. It resembles Bruns' concept of a global polyhedron. The satellite gravimetry missions CHAMP, GRACE and GOCE resulted in a substantial improvement of the global knowledge of the Earth's gravity potential. Its combination at short spatial scales with terrestrial and altimetric gravity anomalies and with topographic heights will allow adding globally consistent gravity potential or height information to the coordinates of the ITRF. In all countries with good geodetic infrastructure an adequate and highly flexible global height system could be established in this manner already now for research and application. It would reveal systematic distortions of existing height systems, identify height offsets between them, and allow it to monitor temporal height changes in a global context. We recommend – as an activity of the Global Geodetic Observing System – the development of a concept of periodical supplementation of the ITRF by globally homogeneous potential/height information.

Zusammenfassung

Der Astronom Heinrich Bruns erörterte 1878 in seiner Denkschrift „Die Figur der Erde" die Bestimmung der Figur der Erde. Ein die ganze Erde umspannendes Polyeder bildet den Geometrieteil. Die Bestimmung des Schwerepotenzials an den Polyederpunkten liefert die notwendige Höheninformation. Bruns zeigte auch, dass die Bestimmung der Erdfigur mit den damals zur Verfügung stehenden Messverfahren theoretisch zwar möglich, praktisch die Realisierung jedoch wegen des Fehlereinflusses der atmosphärischen Refraktion auf die gemessenen Zenitdistanzen und dem Fehlen jeglicher geodätischer Information im Bereich der Weltmeere stark eingeschränkt gewesen wäre.

Die geodätischen Raumverfahren haben die Geodäsie revolutioniert. Durch die Kombination der Raumverfahren Interferometrie über lange Basislinien, Satellitenlaserabstandsmessung, Globale Navigationssatellitensysteme und Doppler Orbitography und Radiopositioning wurde der Geometrieteil des Bruns'schen Polyeders in Form des International Terrestrial Reference Frame (ITRF) bereits Wirklichkeit. In Abständen von ca. fünf Jahren werden die Koordinaten und Geschwindigkeiten einer großen Anzahl von Stationen veröffentlicht. Mit den gravimetrischen Satellitenmissionen CHAMP, GRACE und GOCE wurde zudem die globale Bestimmung des Erdschwerepotenzials entscheidend vorangetrieben. In Kombination mit terrestrischen Schwereanomalien, Schwereanomalien aus Altimetrie und topographischen Höhen ließe sich bereits heute ein relativ genaues globales Höhensystem realisieren. Es würde systematische Verformungen bestehender Höhensysteme offenbaren, Sprünge zwischen den Höhensystemen aufzeigen und ein globales Monitoring von zeitlichen Veränderungen der Höhenkoten ermöglichen. Wir schlagen daher vor, im Rahmen der Arbeiten des Global Geodetic Observing System ein Konzept für die periodische Ergänzung des ITRF durch global einheitliche Potenzial- bzw. Höheninformation zu formulieren.

Keywords: height system, Heinrich Bruns, Bruns' polyhedron, figure of the Earth, terrestrial reference system, geopotential numbers, height datum

1 Introduction

Gravity potential differences between terrain points are a unique and physically meaningful measure of physical height. Traditionally, they are derived from a combination of leveled height increments and gravimetry. This method is very accurate and has been successfully applied in geodesy since more than 120 years (Bomford 1980, ch. 3). If potential differences are referred to an adopted initial zero value at a particular datum point, usually the mean sea level at a selected tide gauge, the potential differences are denoted as geopotential numbers. Geopotential numbers can easily be converted to orthometric or normal heights (Heiskanen and Moritz 1967, Heck 2004). Geodetic government agencies provide physical heights to practice and science. Already at the time of the "Mitteleuropäische Gradmessung" (Central European arc measurements) attempts were made to

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compare mean sea level at selected tide gauges in Europe by means of transcontinental precision leveling networks (Seibt 1883, Börsch et al. 1891). However, classical geodetic leveling has disadvantages, such as:

- the technique is very labor-intensive and therefore expensive and time consuming,
- as the realization of extended height networks takes a long time, it is not easy to associate temporal height changes with a specific epoch,
- generally, leveling is prone to systematic errors,
- the small redundancy in height network adjustments makes it difficult to identify systematic errors,
- each regional height system refers to a regional datum point, usually the mean sea level at a reference tide gauge, in some cases even to more than one datum point. Consequently, there are offsets between datum points. The size of the offsets is of the order of magnitude of the deviation of mean sea level from one common equipotential surface, typically up to ±1 m.

In the eighties and nineties of the twentieth century, with the emergence of GPS, "GPS-leveling" came up as an alternative way of height determination. In the case of GPS-leveling physical heights are deduced from the difference of ellipsoidal heights (from GPS) and geoid heights or height anomalies (from a gravity model). Over short distances, classical leveling is still more precise than GPS-leveling. However, none of the disadvantages of classical leveling listed above applies to GPS-leveling. With GPS-leveling, the realization of a unified global height system comes within reach (Gruber et al. 2012).

The concept of GPS-leveling has been studied, e.g., by Colombo (1980), Rummel and Teunissen (1988), Rapp and Balasubramamia (1992), Rummel and Heck (2001) and Heck (2004). Since the time of these publications, the conditions for GPS-leveling have profoundly improved. This is on the one hand due to the extension and refinement of global navigation satellite systems (GNSS) comprising first the US-GPS alone and meanwhile also the Russian GLONASS, the European Galileo and the Chinese Beidou, and, on the other hand, to the improvement of the geoid part as a result of the satellite gravimetry missions CHAMP, GRACE, and GOCE. The International Association of Geodesy (IAG) looked thoroughly into the potential of this new approach and its implementation for the establishment of a global height system (Ihde and Sànchez 2005, Sànchez 2012, Ihde et al. 2017). Geodesy is concerned with the determination of the geometry of the figure of the Earth, the Earth's rotation and its gravitational field, including all temporal variations. In all three fields – geometry, Earth rotation and gravity – progress of the past few decades was enormous, mainly resulting from new satellite techniques. Subsequently, we propose their combination, with the goal of establishing a geodetic world system of utmost quality. The system would correspond to that described in 1878 already by Heinrich Bruns.

2 The Figure of the Earth according to Bruns

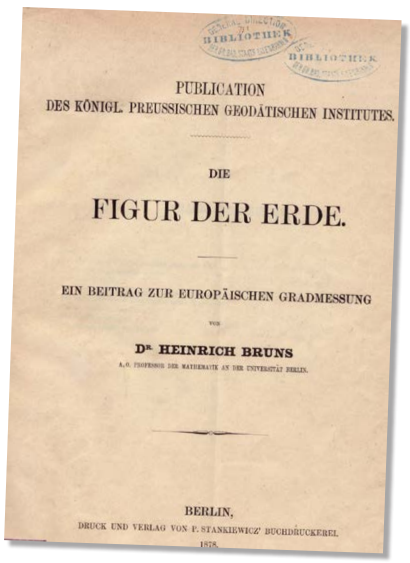
Heinrich Bruns (Fig. 1) published the memorandum "Die Figur der Erde" (The Figure of the Earth) (Bruns 1878), title page in Fig. 2, at the time when the "Mitteleuropäische Gradmessung" (Central European Arc measurement) initiated by general Jacob Baeyer was already underway for several years (Baeyer 1861), see also (Torge 2017). Geodesy became an independent scientific discipline and several renowned scientists took interest in this field. On only 49 pages, Heinrich Bruns succeeded in providing a complete and accurate description of the determination of the figure of the Earth.

Fig. 1: Heinrich Bruns (1848–1919)

At the time of Heinrich Bruns the following five measurement techniques were available to geodesy:

- astronomical positioning with latitude, longitude and azimuth,
- zenith angles,
- horizontal angles and measurement of the length of a baseline,
- geodetic leveling, and
- gravimetry.

Bruns showed that these five techniques were in theory necessary and sufficient for the determination of (1) the figure of the Earth, represented by a global polyhedron, (2) physical heights in the form of gravity potential values at the vertices of the polyhedron and (3) the orientation of the polyhedron relative to the rotation axis of the Earth. He also pointed out the actual limitations of such an approach. First, there was the effect of tropospheric refraction on zenith angles resulting in unacceptably high uncertainties of the geometry of the polyhedron in vertical direction. Second, the oceans were not accessible to any geodetic measurements at Bruns' times.



PUBLICATION
DES KÖNIGL. PREUSSISCHEN GEODÄTISCHEN INSTITUTES.

DIE
FIGUR DER ERDE.

EIN BEITRAG ZUR EUROPÄISCHEN GRADMESSUNG
VON
Dr. HEINRICH BRUNS
A.O. PROFESSOR DER MATHEMATIK AN DER UNIVERSITÄT BERLIN.

BERLIN,
DRUCK UND VERLAG VON P. STANKIEWICZ' BUCHDRUCKEREI.
1878.

Fig. 2: Front page of the memorandum "Die Figur der Erde" (The Figure of the Earth)

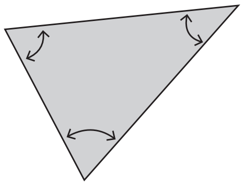


Fig. 3.1: Bruns' polyhedron starts with one triangle of a larger triangulation network. The triangle connects three adjacent terrain points.

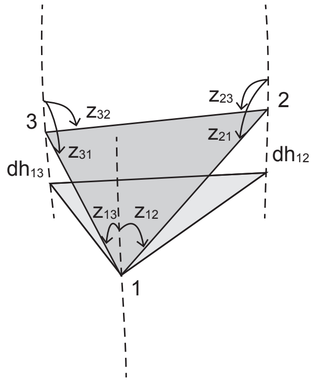


Fig. 3.2: Zenith angles at the vertices of the triangle, together with one side length give the height differences between the vertices and allow the projection of the triangle on a reference surface.

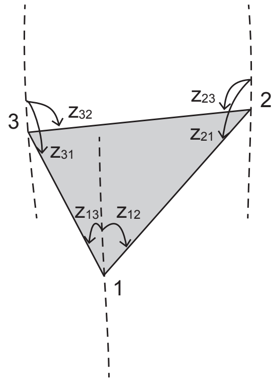


Fig. 3.3: The zenith angles fix the directions of the plumb lines relative to the three sides. The shape of the triangle is fixed by the measured elements; it can still be freely shifted and rotated. It is a geometric form element (Baarda et al. 1956).

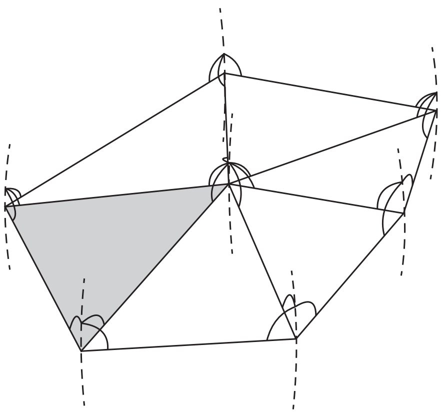


Fig. 3.4: With a whole set of triangles a terrain segment is represented. Adjacent triangles share one side together with the plumb lines at its ends.

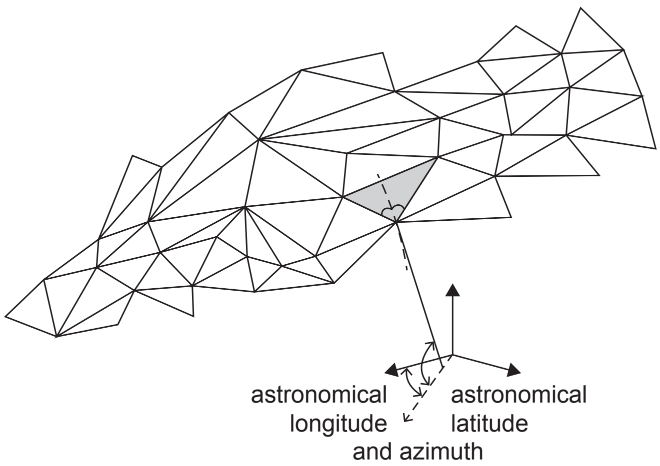


Fig. 3.5: Astronomical positioning at several points allows to fix the position and orientation of the network (= geometric form element) with respect to the Earth's axis and Greenwich meridian plane (Strang van Hees 1982).

Consequently, the polyhedron approach would consist of several patches confined to land areas. Each of these patches would have a separate horizontal and height datum, the latter usually represented by mean sea level at a coastal tide gauge. This would result in unknown height offsets between the various height systems. On page 5 of his memorandum, he discusses their causes: the deviation of mean sea level from one common equipotential surface of the gravity field due to atmospheric (wind, pressure) and oceanic effects (ocean currents).

How would the realization of Bruns' polyhedron have looked like at his time? The steps are illustrated by Figs. 3.1 to 3.5: One triangle of a triangulation network would be the point of departure. The triangle connects three adjacent terrain points. Zenith angles and the endpoints of a baseline allow the computation of its projection onto a horizontal plane. The zenith angles at the three vertices of the triangle, together with the side lengths, give the geometric height differences between the vertices. The zenith angles fix the direction of the plumb line relative to the three sides. Together, this procedure constitutes a geometric form element in the sense of Baarda et al. (1956). The shape of the triangle is fixed by the measured elements; it can still be shifted and rotated in space. With a whole set of triangles a terrain segment is represented. Adjacent triangles share one side, together with the two plumb lines at its endpoints. The elementary triangle together with the plumb lines at its vertices extends to a much larger geometric form element, following the shape of the terrain, still leaving undefined the degrees of freedom of translation and rotation. Adding astronomical positioning at several points determines the orientation of the larger form element with respect to the Earth's axis and to the Greenwich meridian plane (Strang van Hees 1982). The distances of the form element from the Earth's axis and the geocenter remain unknown. Following this procedure, a global polyhedron can be constructed, in principle. The necessary physical height information (information about points being higher, or lower, or at the same height) is obtained by adding to the purely geometrical global form element, i.e., Bruns' polyhedron, gravity potential differences between all vertices. They are calculated from leveled height differences and measured gravity. Referring the gravity potential to mean sea level via the introduction of a zero datum value, geopotential numbers are obtained. Bruns also showed how to determine a gravity potential reference value W_0. As already mentioned, the basic obstacles were on the one hand the oceans, and on the other hand the effect of tropospheric refraction on the measurement of zenith angles.

3 Bruns' polyhedron today: ITRF combined with a high resolution geopotential model

Bruns' "Die Figur der Erde" has been published in 1878, i.e., 140 years ago. Since then the tools of geodesy have improved significantly. The past 70 years have been remarkable indeed: electronic measurement techniques emerged together with modern computing. Most importantly, space age resulted in a quantum leap for geodesy. Today the determination of the figure of the Earth are based on four complementary space techniques:

- Very Long Baseline Interferometry (VLBI),
- Satellite Laser Ranging (SLR),
- Global Navigation Satellite Systems (GNSS), and
- DORIS, the satellite-based French orbit determination system.

For each of the four techniques, a scientific service has been established under the umbrella of IAG, collecting the observations and processing them according to common standards. They are combined to result in the International Terrestrial Reference System (ITRS), (Petit and Luzum 2010). The IERS coordinates this work. The ITRF is the realization of the ITRS. The ITRF is essentially a register of coordinates of terrain points together with their standard deviations and station velocities. The processing of the ITRS/ITRF is rather complex and includes a large number of effects such as solid-Earth and ocean tides, atmospheric and ocean loading and plate motion. At intervals of approximately five years, new releases of the ITRS/ITRF are produced (Altamimi et al. 2011, Altamimi et al. 2017, Seitz et al. 2017). The coordinates are given in a geocentric coordinate system. The coordinate precision is a few millimeters. The list of about 1500 stations of the ITRF resembles rather well a Bruns' polyhedron. Nowadays, a hierarchy of national systems, mostly based on GNSS-measurements, densifies and complements the ITRF. To some extent, tropospheric refraction remains a weak point. However due to the measurement geometry from the GNSS satellites to stations on the Earth's surface, the error standard deviations of the vertical coordinates are only 1.5 to 2 times higher than those of the horizontal ones (Meindl 2011). Today, the oceans do no longer represent an obstacle to geodesy. In parallel to the activities related to the ITRS/ITRF, the geometry of the sea surface is monitored with centimeter precision since already almost thirty years (Fu and Cazenave 2001, Stammer and Cazenave 2017) by satellite altimetry. In summary, owing to modern geodetic space techniques, the geometry part of Bruns' polyhedron is reality.

How can the excellent 3D geometry be complemented with the required height information, or, following Bruns, how can the gravity potential be added and – if convenient – gravity and deflection of the vertical? In parallel to the enormous progress in GNSS positioning and satellite altimetry, a similar success story started in the field of satellite gravimetry. GNSS on low Earth orbiting satellites (LEO) permits uninterrupted and very accurate 3D tracking of their orbits around the Earth, or, from the point of view of gravity field research, of the free fall of the LEOs' around the Earth in our planet's gravitational field. Furthermore, highly precise accelerometers on-board the satellites , or, alternatively, the monitoring of highly accurate distance variations between satellites, allow, dependent of the chosen strategy, either the measurement of the effect of non-gravitational forces on the LEOs' trajectories or of the micro-gravitational field in the LEOs' interior relative to its center-of-mass. The satellite missions CHAllenging Minisatellite Pay-load (CHAMP, 2000–2010), Gravity Recovery And Climate Experiment (GRACE, 2002–2017) and Gravity and steady-state Ocean Circulation Explorer (GOCE, 2009–2013) applied these principles. From each of them and from their combination, a new quality of global gravity modeling emerged, both in terms of spatial resolution and of precision. The models are usually given as complete coefficient sets of a spherical harmonic series up to a maximum degree and order (d/o). Examples are the model TIM-5 up to d/o 280 derived solely from GOCE data, or the model DIR-5 up to d/o 300, based on data from GRACE, GOCE, LAGEOS-1 and LAGEOS-2 (Brockmann et al. 2014, Bruinsma et al. 2014).

The maximum degree of the spherical harmonic series corresponds to the spatial resolution on the Earth's surface. In a simplified form, the following rule-of-thumb can be used:

Spatial resolution [km] = 20,000 km/maximum degree.

Thus, d/o 280 corresponds to scales down to 71 kilometers and d/o 300 to about 66 kilometers. In both cases, the resulting precision of the gravity potential corresponds to a height precision between 1 cm and 3 cm. With either of these two models, the 3D coordinates of the stations of the ITRF can thus be complemented with height information (geopotential numbers, orthometric or normal heights) with a height precision of 1 cm to 3 cm at this maximum d/o. The proposed procedure follows the idea of Heinrich Bruns, i.e., the geometric description of the figure of the Earth in the form of a global polyhedron is complemented by the necessary height information in the form of gravity potential information. According to Marussi (1985) this procedure is synonymous to adding so-called natural or gravity coordinates to the geometric coordinates of the polyhedron. In another work, Marussi (1977) speaks of a "geometry inside the geometry".

Unfortunately, one fundamental limitation remains. The truncation of the spherical harmonic series at a maximum degree, e.g., at degree 300 as in the case of DIR-5, results in a smoothed and therefore incomplete representation of the gravity field – and consequently of the physical heights derived from it. The neglected part of the field is referred to as truncation error in the spectral domain. Assuming, for example, a series expansion of the geoid up to d/o 200, the degree-variance model by Tscherning and Rapp (1974) predicts a truncation error of typically 48 cm, the simpler model by Kaula (1966) one of about 28 cm; both values would be unacceptably high for a national height system. The truncation error has to be understood as a global standard deviation, with regions of higher and of lower values, depending on the roughness of the terrain. The truncation error can be reduced significantly by amending the satellite gravity model with terrestrial data. There are regions with very good and complete terrestrial gravity data and others with data gaps, classified or poorly documented data. The most obvious approach for the realization of a global height system is therefore the method of GPS-leveling combining precise ellipsoidal heights of points of the ITRF with gravity potential information derived from a so-called high-resolution combined gravity model. Such high-resolution gravity models adopt a rather complex and rigorous functional model for the combination of satellite gravimetry and a worldwide collection of terrestrial gravity anomalies, in ocean areas of altimetric gravity anomalies, and in areas lacking gravity data fill-in information of gravity deduced from topographic data. The EGM2008 (Pavlis et al. 2012) is an excellent example of a model of this kind. It consists of a set of spherical harmonic coefficients up to d/o 2159, with additional terms up to 2190. More recent models are GOCE05c-ogmoc of the GeoForschungsZentrum (GFZ) in Potsdam, also up to d/o 2190, and GOCO05c as well as XGM2016, with a direct solution of the joint – satellite and terrestrial – system of normal equations up to d/o 719 (Fecher et al. 2017, Pail et al. 2018). The remaining truncation error for the latter two models is about 8 cm to 9 cm, while it is only about 2 cm in the case of the high-resolution combined model. Thus, also the gravity potential – or height – part of the idea of Heinrich Bruns can be realized today, in principle, albeit not yet everywhere with the desired precision. Height determination in the above sense would fit very well into the activities of the Global Geodetic Observing System (GGOS) of IAG (Beutler and Rummel 2012).

We therefore propose the realization of a globally consistent height system following the concept of Heinrich Bruns. The key elements are:

- Realization of the globally unified height system through the addition of gravity potential values (and/or geopotential numbers, normal heights, orthometric heights) to the 3D station coordinates of the ITRF. This gravity potential or height part of the Bruns polyhedron could be complemented by the first derivatives of the potential, i.e., the gravity disturbances and deflections of the vertical in a local {north, east, up}-triad.
- The ITRF also comprises reference markers of national time services. This is done for the maintenance of the time scales of the GNSS. In view of the rapidly increasing precision of optical clocks, it may soon be possible to use them for the determination of precise gravity potential differences between reference clocks. Consequently, markers of the national time services may get an additional role as control and reference points of the global height system (Müller et al. 2017, Denker et al. 2018).
- The extension of the station coordinate list of the ITRF by a selection of national terrestrial reference points and tide gauges. This extended list would consist of 3D coordinates and the gravity potential/height information, as well. The complete set of station coordinates may be referred to as Bruns' polyhedron.
- The catalogue of station heights would include geometric heights (heights above an adopted reference ellipsoid) and physical heights (or geopotential numbers) as well as their temporal changes.
- GGOS-IAG would be responsible for the collection, assessment, and processing of gravity and height data as input of a next generation high-resolution gravity model, the model succeeding EGM2008 and GOCO05c-ogmoc. The process includes the assessment of the theoretical and numerical characteristics of such a model including its compatibility with the ITRS.
- GGOS-IAG would also be responsible for periodic releases and updates of the gravity potential/height part.

The implementation of these recommendations requires the study of a series of open issues. As an activity of the IERS the ITRS/ITRF is continuously refined and up-dated (Altamimi et al. 2011, Altamimi et al. 2017, Seitz et al. 2017). It would be only logical if the same type of up-date and refinement would take place in parallel for the gravity potential/height part.

If the gravity potential/height part is based on the best possible and most complete high-resolution combined gravity field model, GGOS should take care of the necessary input data. After GRACE, GOCE, and the new computation of oceanic gravity anomalies based on CRYOSAT-2 (Andersen and Knudsen 2016), the focus has to be on terrestrial gravity anomalies. Similar to the very successful campaigns in the Arctic and the Antarctic (Kenyon et al. 2008, Scheinert et al. 2016) IAG could support improvement of the data situation in other regions with poor coverage or gaps.

Because physical heights enter into the computation of terrestrial gravity anomalies, these are affected by errors due to the unknown height offsets between the various height datum zones. Rummel and Teunissen (1988) showed how to deal with this aspect when solving the geodetic boundary value problem. Xu (1992) carried out corresponding numerical tests. Gerlach and Rummel (2012) demonstrated that this indirect datum related gravity anomaly error is negligible due to the use of the new generation of excellent satellite-based gravity models. The complete consistency of the geometry part with the gravity potential/height part is of great importance, i.e., the coordinates of the stations of the ITRF have to be consistent with the gravity potential/height part. This requirement implies the necessity of applying the same fundamental parameters and reduction models for both parts. Gerlach et al. (2017) discuss the relevant issues, such as one common system origin of ITRS/ITRF (close to the Earth's center of mass) and gravity potential (center of mass by definition), temporal changes of the figure of the Earth and of the Earth's gravity field, and the issue of permanent tides (Hughes and Bingham 2008, Mäkinen and Ihde 2009). In front of the background of climate research and ocean applications, it is worthwhile to re-assess the IAG recommendation concerning the preference of adopting the zero-tide system (IAG 1984, Petit and Luzum 2010). Would it not be more operational to work in the mean-tide system? The result would be a model of the Earth's gravity field with a geoid surface much closer to mean sea level – as deduced from tide gauge records and satellite altimetry.

A word on the height datum definition and the determination of W_0: Bruns (1878) addressed this aspect, probably because of his deep interest and great expertise in potential theory. We also refer to (Heiskanen and Moritz 1967, secs. 2-19 and 2-20; Rummel and Heck 2001, Hipkin 2001 and Sacerdote and Sansò 2001) in this context. In analogy to electrostatics, only gravity potential differences can be measured, not the potential itself. This poses no problem and is no limitation, because in any application only potential differences are needed. In geodetic theory, in the context of the solution of the geodetic boundary value problem, the (absolute) potential is made an estimable quantity through the introduction of the regularity condition. The condition assumes the potential to converge towards zero at infinity. The gravity potential at the Earth's surface becomes estimable with this boundary condition. Consequently, also in this case when one deals with potential differences, implicitly, namely with the difference of the potential at infinity and at a point on the Earth's surface. It is, however, still necessary to define a height datum. The height datum defines the one among all possible equipotential surfaces to which the geopotential numbers/heights refer. The choice could be, e.g., the level surface coinciding with mean sea level at a tide gauge of maximum tectonic stability, or the ensemble mean value of a selected set of tide gauges, or the mean of the grid values of the global altimetric mean sea level. Any of these datum selections would allow it to switch from one datum definition to another one via one-dimensional S-transformation (Baarda 1973). Investigations concerning the best possible W_0 should be understood in this sense (Burša et al. 1997, 1999, 2007; Grafarend and Ardalan 1997).

A further point of consideration is the optimal use of the strength of existing classical height data (leveling combined with gravity) in the proposed process. It is well-known that leveling is extremely precise over short distances while systematic errors may accumulate over longer distances, resulting in significant distortions of existing continental and national height systems, as discussed, e.g., by Higginsen et al. (2015), Wang et al. (2012) and Woodworth et al. (2012). The old literature (Bomford 1980; Jordan, Eggert and Kneissl 1956, pp. 218–264) discusses the sources and the characteristics of systematic leveling errors. A procedure needs to be designed which combines the strength of GPS-leveling over long distances with that of classical leveling over short distances.

4 Outlook

In 1878 Heinrich Bruns discussed a theoretical concept to determine the figure of the Earth. He proposed to represent the Earth's figure by a global polyhedron with gravity potential values at its vertices as the unique physical measure of height. By analyzing all available measurement techniques, he concluded that a realization was impossible at his time. Zenith angles were severely affected by atmospheric refraction and geodetic measurements were confined to land areas. Since the advent of space age in 1957, these limitations disappeared. Seventy years after the launch of Sputnik-1, the determination with centimeter precision of the geometry of the figure of the Earth is reality. Geodetic satellite techniques are available for land areas as well as for ice and ocean regions.

What about the gravity potential/height part? Our knowledge of the Earth's gravity field and the geoid has greatly improved with the gravimetric satellite missions CHAMP, GRACE, and GOCE. In the case of GRACE and GOCE, a special effort was made to counteract the natural field attenuation. GOCE combined an extremely low orbital altitude with gravity gradiometry. GRACE consisted of two identical "free falling" satellites following each other in the same low Earth orbit, their distance variations being measured with micrometer precision. Now we have gravity models with high precision, global consistency, and rather high spatial resolution. With the objective to continue the measurement series of GRACE and possibly improve the spatial and temporal resolution, GRACE Follow-on was launched on 22.5.2018 (Flechtner et al. 2018). Still, for height determination with centimeter precision these models need to be amended with terrestrial and ocean gravity and topographic data in order to utilize them for the gravity potential/height part of Bruns' polyhedron.

The realization of a consistent and global height system is an important part of the geodetic mission since the time of Heinrich Bruns. Today, with precise GNSS and with very accurate, high-resolution gravity potential models, a unified global height system can be realized. Very likely, it will not – yet – meet geodetic standards everywhere on the globe. It would, however, be the starting point of a unified global height system and allow it to reveal systematic distortions and height offsets of existing regional height systems. It would serve major engineering projects and be another important contribution of geodesy to sea level change and climate research.

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